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"SOME OBSERVATIONS ON THE BEHAVIOR

OF THE LANGLEY MODEL ROTOR BLADE"

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INTRODUCTION

In our statement of work for this grant, the first item is "support and coordinate with research underway at the Aerostructures Directorate ...." This interim report supports the design of the model rotor and the comparative study of coupled beam theory and the finite element analysis performed earlier at the Aerostructures Directorate by Robert Hodges and Mark Nixon.

Attention is focused upon two matters --- (1) an examination of the small discrepancies between twist angle predictions under pure torque and radial loading and (2) an assessment of nonclassical effects in bending behavior.

Our primary objective is understanding, particularly with regard to cause-effect relationships. Understanding, together with the simple, affordable nature of the coupled beam analysis, provides a sound basis for design.

STATIC APPLIED LOADING CASES

The three load cases considered by Hodges and Nixon have been considered here. The first case is bending due to lift and blade weight, the second is pure torque and the third is axial loading due to centrifugal force.

There is some inconsistency in the equations for the applied loading. In the present work, the coordinate X is taken from the blade root, which is radial station 5.23.

Bending Due to Lift and Blade Weight

The distributed loading is

$$q_z = 0.02222X - 0.0123 \text{ (lbs/in)} \qquad (1)$$

The rotor model cross section appears in Figure 1. The coupled beam analysis of this loading case appears in Attachment 1.

Beam deflection results appear in Figure 2. Bernouli-Euler, the classical engineering beam theory, results are denoted by "BE." This model is overly stiff. Also presented are three shear deformation models, SD1, SD2 and SD3, and the finite element results.

The shear deformation model S1 is an approximation obtained by setting the coupling stiffness C_{25} and C_{36} to zero. This is the classical shear deformation model in the spirit of Timoshenko. Clearly it is overly stiff also. This direct transverse shear effect is small for a beam of this slenderness.

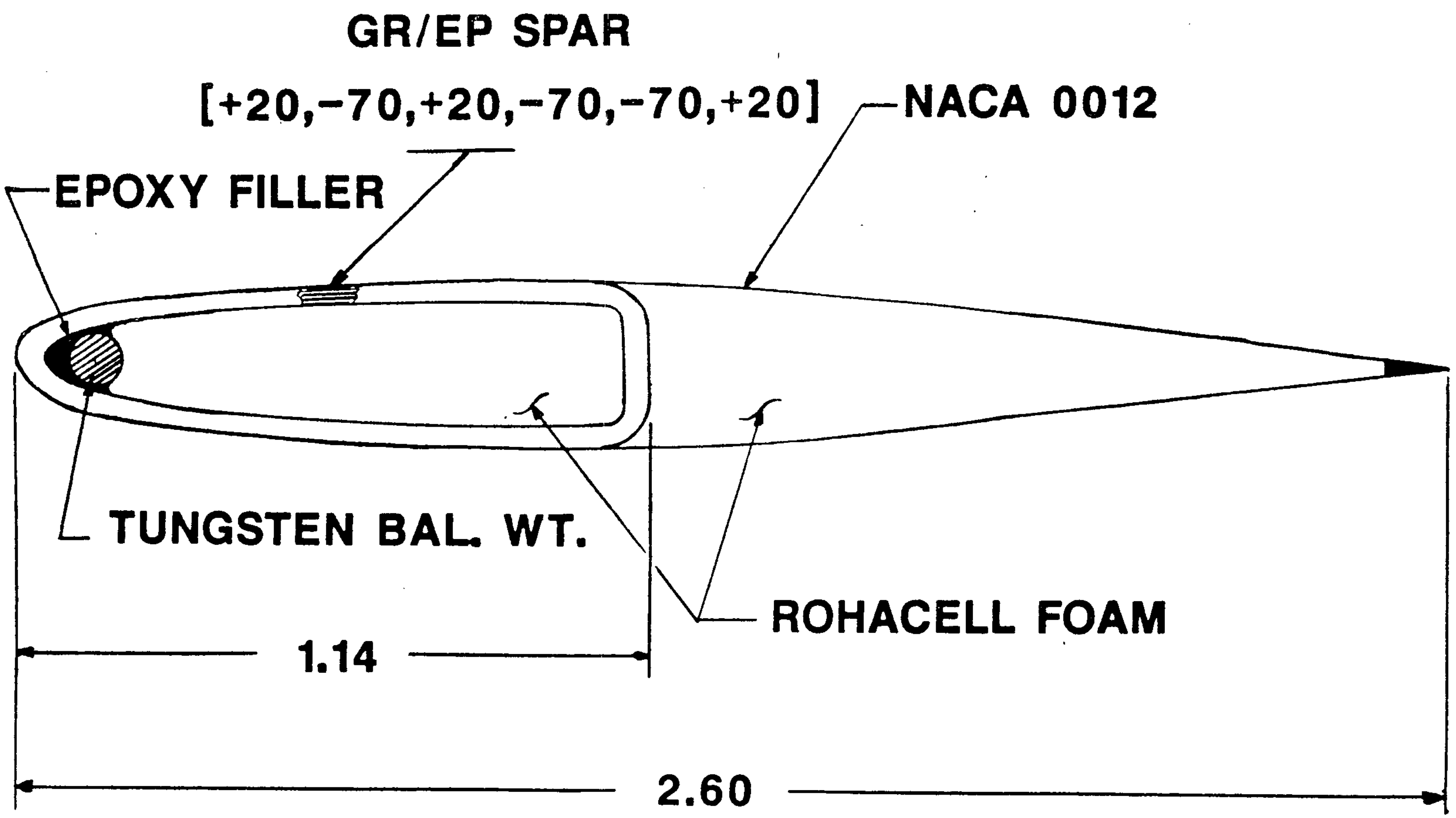
The complete theory, which includes all coupling effects, is denoted SD3. It provides good agreement with the finite element results.

The approximation denoted SD2 is obtained by neglecting completely the classical shear deformation effect accounted for in SD1 in favor of the coupling mechanism associated with C_{25} and C_{36}. This model, therefore, includes only deformations due to the transverse shear-bending coupling and the usual bending contribution. The magnitude of this new, unexplored form of elastic coupling is seen to be enormous by comparing SD2 and BE results. This is a finding of major importance in understanding the behavior.

The SD2 or SD3 models are required in this application in order to get sufficiently accurate predictions. This clearly excludes the earlier classical type theory of Mansfield and Sobey from practical use.

FIG. 1

MODEL ROTOR CROSS SECTION



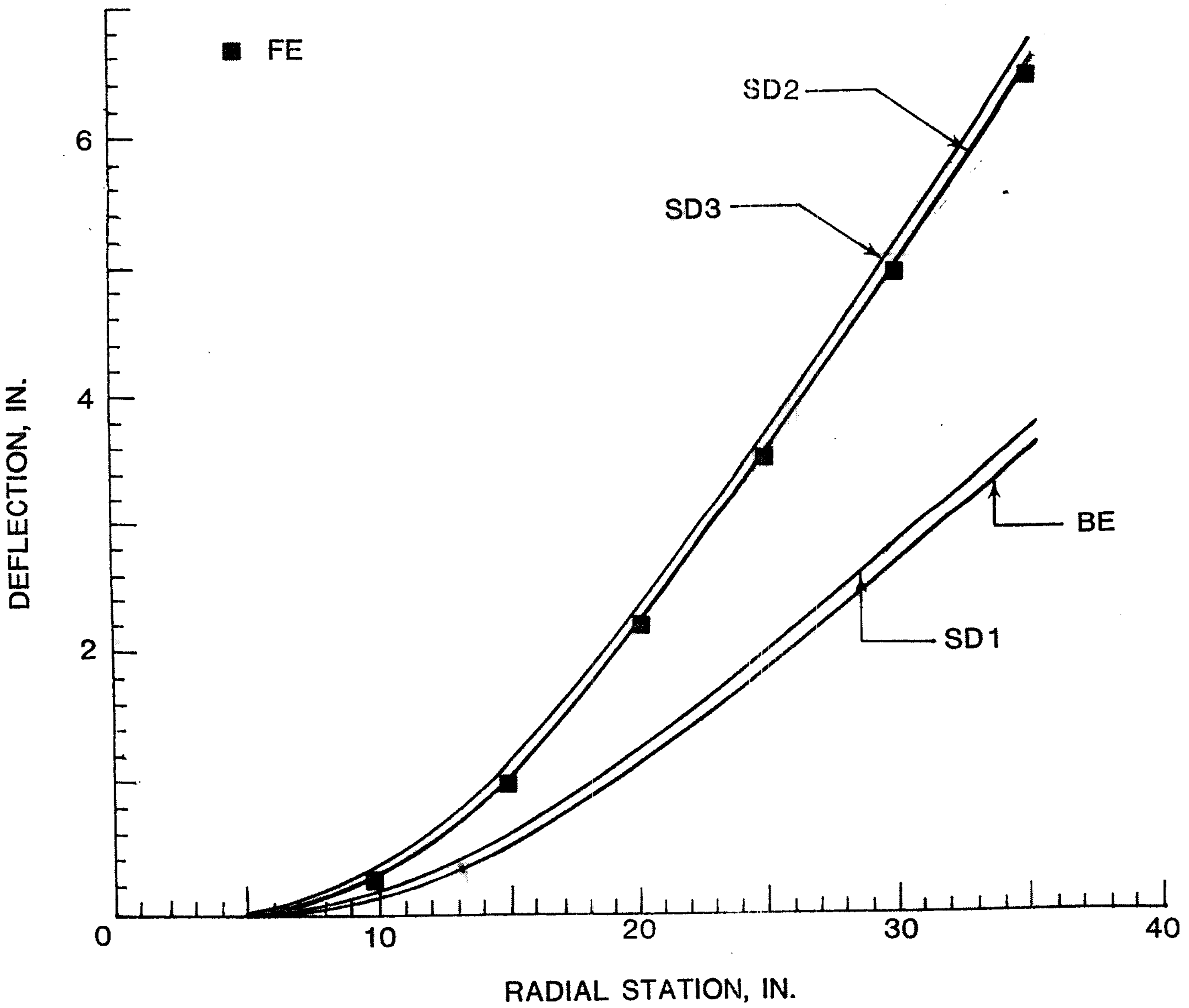


FIGURE 2

BEAM DEFLECTION DUE TO LIFT AND BLADE WEIGHT

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Pure Torque

Although there was generally good agreement for the torsion case in the Hodges-Nixon comparative study, the effect of torsion-related warping was not included. This effect has been included in the analysis presented in Attachment 2.

The classical St. Venant torsion theory result (without warping) is compared to the complete beam theory (CBT) and the finite element results in Figure 3. The CBT results, which differ from the classical (CL) only by the warping effect, are in excellent agreement with the finite element analysis. Restrained warping creates a boundary layer zone near the blade root that acts to stiffen the blade and reduce the angle of twist.

Axial Loading Due to Centrifugal Force

This case is of the utmost importance because extension-twist coupling is to be used to control blade stall. In the Hodges-Nixon comparative study, the classical St. Venant theory was utilized for the coupled beam analysis. The discrepancy between analytical predictions and the finite element analysis was the greatest for this case. Classical theory was too soft and it overestimated the twist angle, a condition that is not conservative in view of the stated purpose of the model demonstration.

As in the pure torsion case, the neglect of torsion-related warping is the reason for the discrepancy between coupled beam theory and the finite element analysis. A complete analysis of this loading case is given in Attachments 3 and 4. Attachment 3 contains the overall response analysis. The axial force distribution is

$$N = 913.83 - 7.875X - 0.75287X^2 \text{ (lb.)} \quad (2)$$

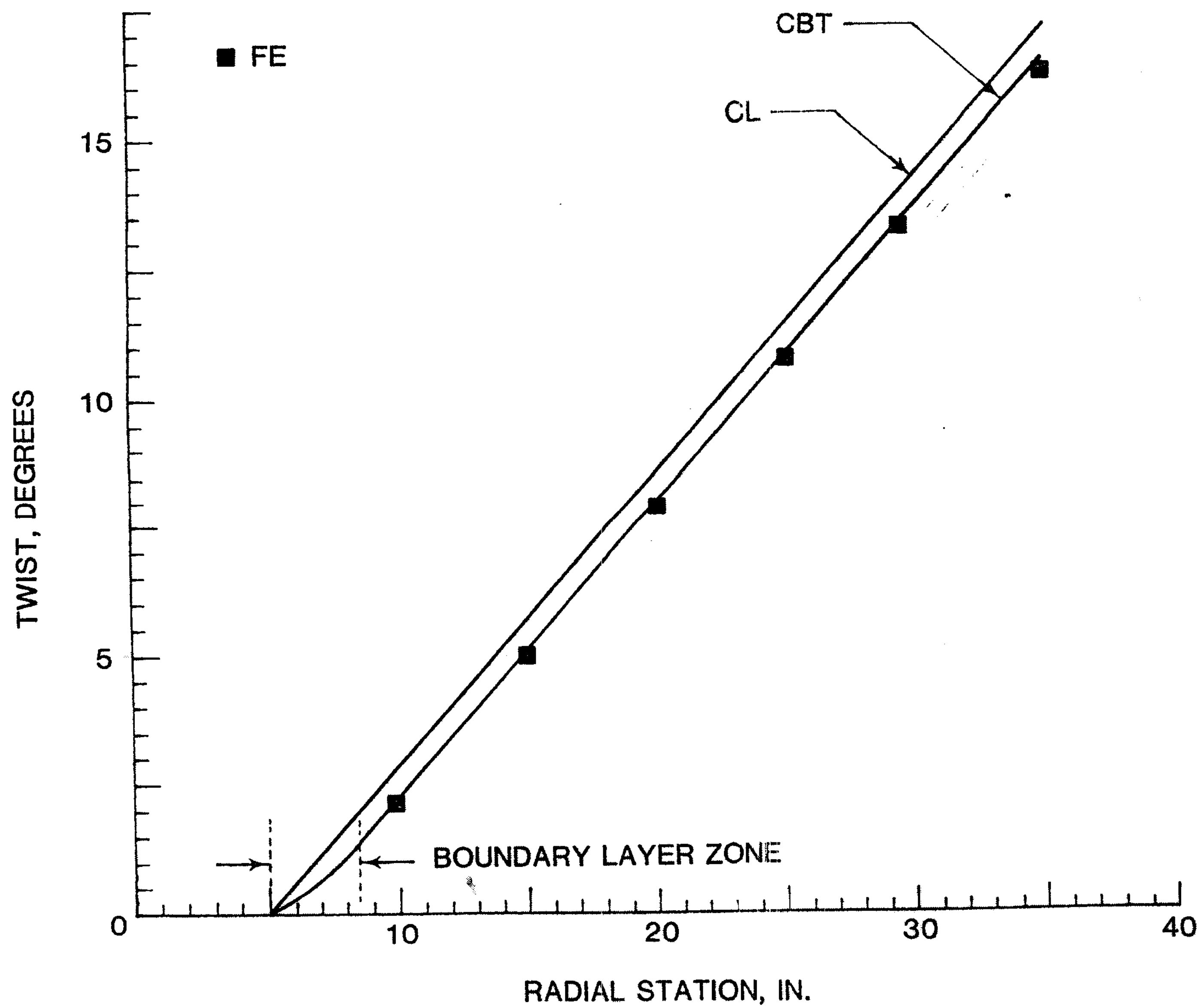

FIGURE 3

TWIST DUE TO APPLIED TORQUE

This expression differs from that quoted in the Hodges-Nixon work due to use of different coordinates.

The twist angle distribution appears in Figure 4. The use of CBT brings the beam theory results in very good agreement with the finite element analysis. The rate of twist distribution is given in Figure 5. Again, the agreement is very good.

Attachment 4 contains an analysis of the strain distributions for this loading case. The strain distributions are given in Figures 6 and 7. The results indicate that structural damage would be likely to occur at radial station 10 ($X \cong 5$) rather than at the root as predicted by classical theory.

WARPING ANALYSIS

A complete analysis of the effects of torsion-related warping appears in Attachment 5. Also included is a description of a simple warping model that is based upon a rectangular approximation for the cell. The equivalent rectangle is chosen to possess the same enclosed area. An assessment of this model suggests that it is adequate for the complete analysis.

The main difficulty in accounting for warping is determination of the warping function and the stiffness C_{77}. Both are accomplished readily with the approximate rectangular model.

CONCLUSIONS

In structures designed for extension-twist coupling, a high degree of bending-shear coupling is present which drastically causes the structure to be more flexible in bending. The impact of this effect on system performance must be assessed.

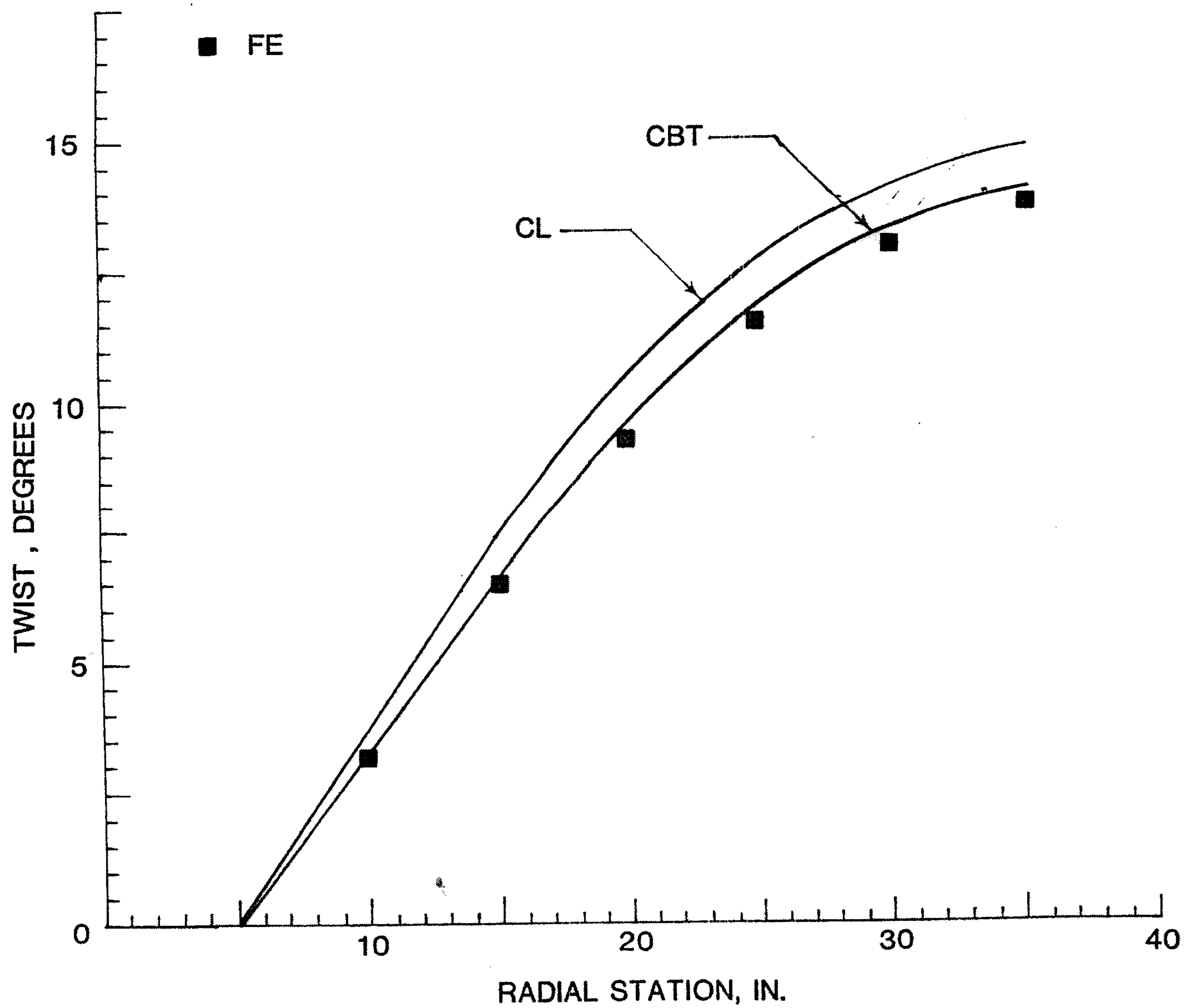

FIGURE 4

TWIST DUE TO CENTRIFUGAL FORCE

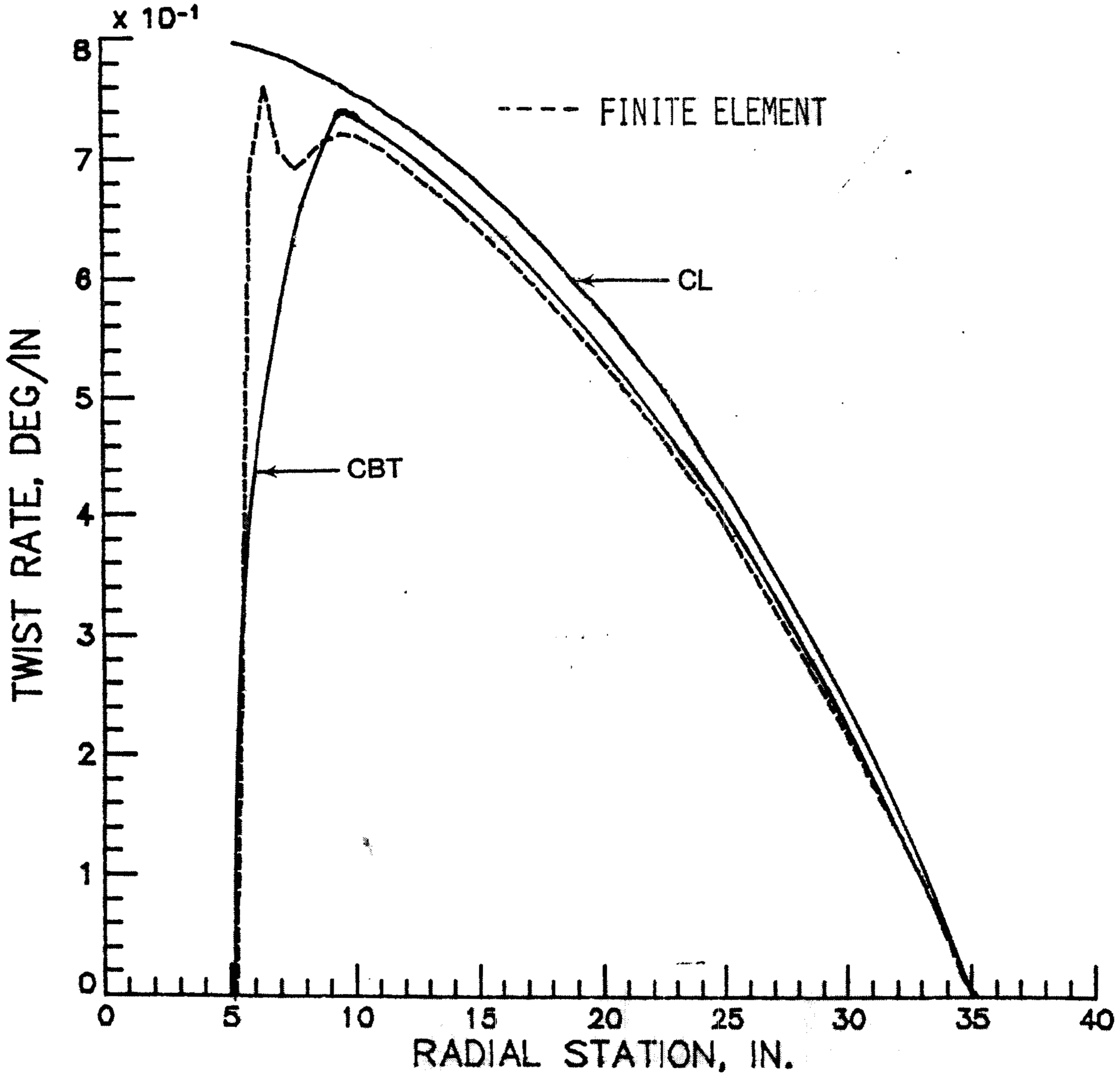

Figure 5. - Twist rate due to centrifugal force.

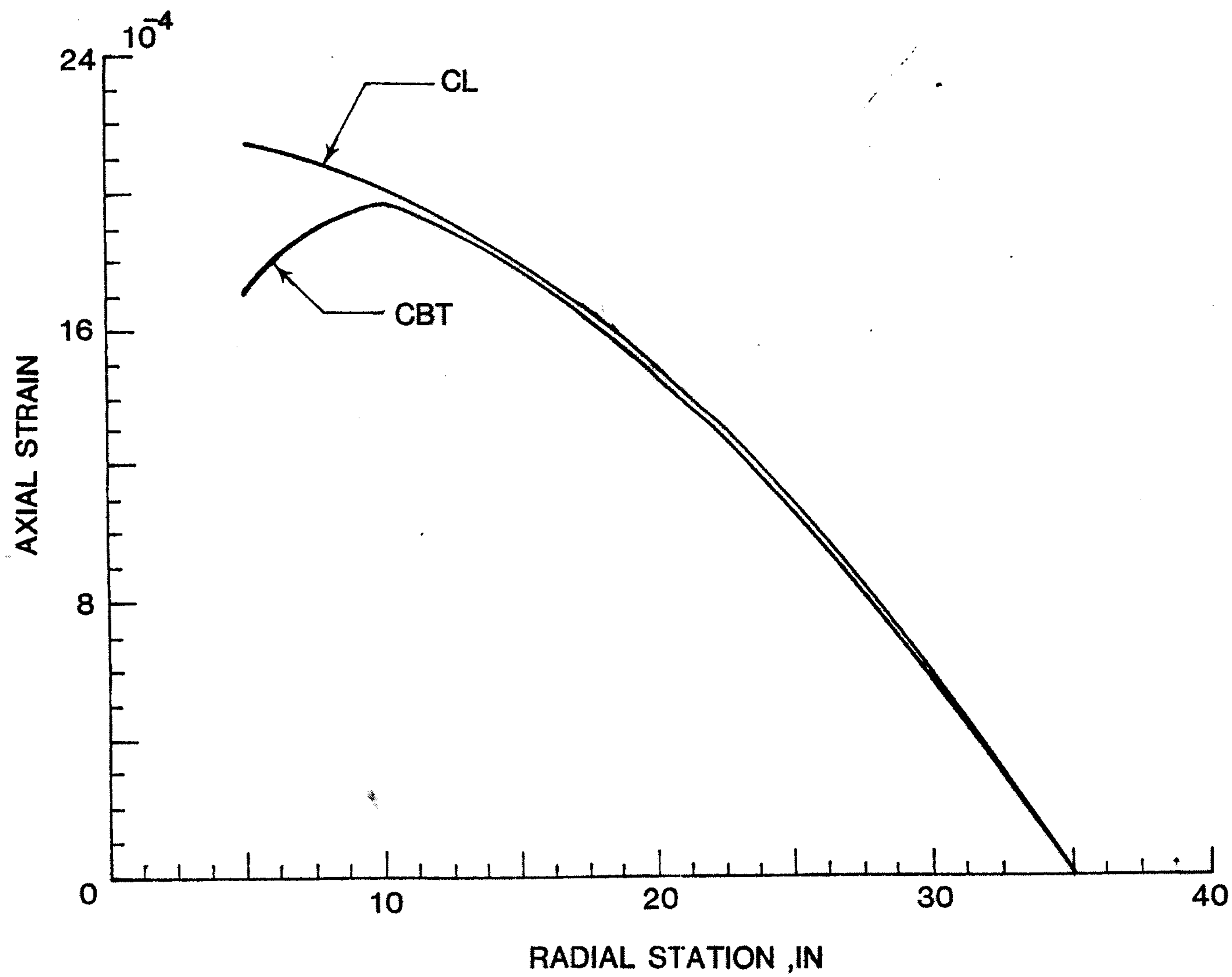

FIGURE 6

AXIAL STRAIN DUE TO CENTRIFUGAL [illegible]

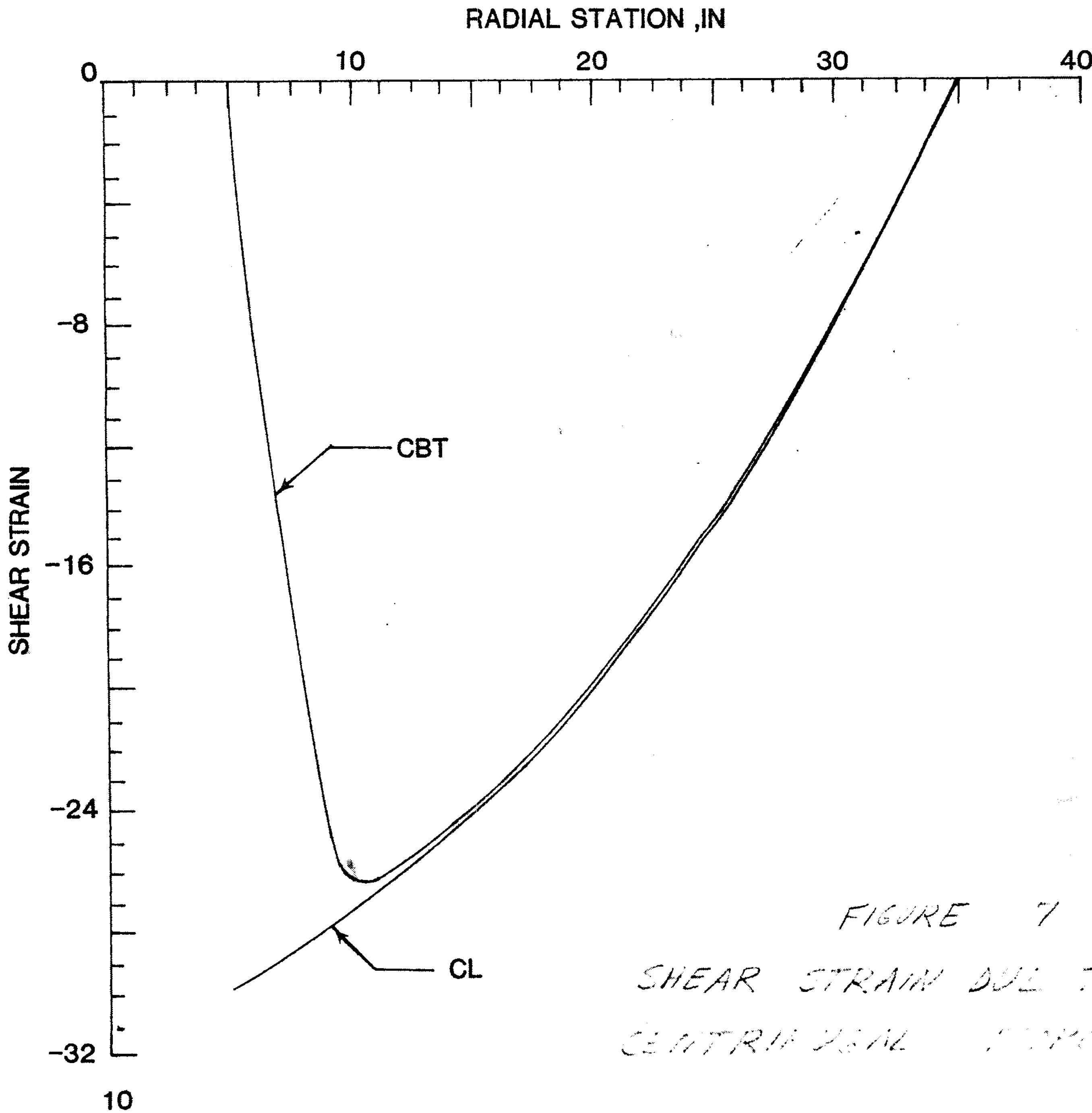

FIGURE 7
SHEAR STRAIN DUE TO
CENTRIFUGAL FORCE

Torsion-related warping is significant enough to warrant its inclusion in the beam analysis. A simple rectangular approximation may be used, which avoids the complexities associated with warping function and warping stiffness determination for sections similar to the D spar. With warping accounted for, the coupled beam theory is extremely accurate and easy to use.

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ATTACHMENT 1

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COMPARISON OF COMPOSITE ROTOR BLADE MODELS : CLASSICAL , CLASSICALSHEAR DEFORMATION , SHEAR DEFORMATION and AN MSC NASTRAN SHELL ELEMENT

A BENCH MARK CASE : BEAM DEFLECTION DUE TO LIFT and BLADE WEIGHT

THEORY

The beam cross section rotation about the y axis is written in terms of beam compliance terms and applied loads

$$\beta_{y,x} = S_{25} Q_y + S_{55} M_y \tag{1}$$

The section rotation about the y axis is also defined in terms of the shear strain and the transverse displacement

$$\beta_y = \gamma_{xz} - W_{,x} \tag{2}$$

Derivation of equation (2) respect to x gives

$$\beta_{y,x} = \gamma_{xz,x} - W_{,xx} \tag{3}$$

The shear strain, γ_{xz}, can then be written in terms of beam compliance terms and applied loads

$$\gamma_{xz} = S_{33} Q_z + S_{36} M_z \tag{4}$$

Derivation of equation (4) respect to x yields

$$\gamma_{xz,x} = S_{33} Q_{z,x} + S_{36} M_{z,x} \tag{5}$$

Substitution of equation (5) into equation (3) gives

$$\beta_{y,x} = S_{33} Q_{z,x} + S_{36} M_{z,x} - W_{,xx} \tag{6}$$

and substitution $\beta_{y,x}$ into equation (1) defines the deflection as

$$-W_{,xx} = S_{25} Q_y + S_{55} M_y - S_{33} Q_{z,x} - S_{36} M_{z,x} \tag{7}$$

In this case

$$Q_y = M_z = 0 \tag{8}$$

therefore equation (7) becomes

$$-W_{,xx} = S_{55} M_y - S_{33} Q_{z,x} \tag{9}$$

Derivation of the shear, $Q_{z,x}$, gives the load P

$$Q_{z,x} = -P \tag{10}$$

so, equation (9)

$$-W_{,xx} = S_{55} M_y + S_{33} P \tag{11}$$

Shear Deformation Model

Rewrite equation (11)

$$-W_{,xx} = S_{55} M_y + S_{33} P$$

where

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$$S_{55} = 1 / (C_{55} - C_{25}^2 / C_{22}) \qquad (11a)$$

and

$$S_{33} = 1 / (C_{33} - C_{36}^2 / C_{66}) \qquad (11b)$$

Classical Shear Deformation Model

In this model, it is assumed that

$$C_{25} = C_{36} = 0 \qquad (12)$$

so, equations (11a) & (11b) become

$$S_{55} = 1 / C_{55} \qquad (13a)$$

$$S_{33} = 1 / C_{33} \qquad (13b)$$

thus equation (11) is refined as

$$-W_{,xx} = \frac{M_y}{C_{55}} + \frac{P}{C_{33}} \qquad (14)$$

Classical Model

In the classical model, it is assumed that shear strain is negligible; therefore equation (8)

$$\beta_{y,x} = -W_{,xx} \qquad (15)$$

and using equation (8)

The deflection [illegible] as

$$-W_{,xx} = \frac{M_y}{C_{55}} \qquad (16)$$

APPLICATION

The load, p, due to lift and blade weight can be written in terms of x as

$$p = 0.02222x - 0.0123 \qquad [\text{lbs/in}] \qquad (17)$$

Integrating once gives the shear.

$$Q_z = -0.01111x^2 + 0.0123x + C_1 \qquad (18)$$

B.C is

$$Q_z\Big|_{x=30} = 0 \qquad (19)$$

so

$$Q_z = -0.01111x^2 + 0.0123x + 9.631 \qquad [\text{lbs}] \qquad (20)$$

Integrating again gives the moment

$$M_y = -0.003704x^3 + 0.00615x^2 + 9.631x + C_2 \qquad (21)$$

B.C is

$$M_y\Big|_{x=30} = 0 \qquad (22)$$

so

$$M_y = -0.003704x^3 + 0.00615x^2 + 9.631x - 194.5 \qquad [\text{lb-in}] \qquad (23)$$

Come back to equation (11); integrating twice with using boundary conditions which are

$$W_{,x}\Big|_{x=0} = 0 \qquad (24a)$$

$$W\Big|_{x=0} = 0 \qquad (24b)$$

given the deflection function as follows

$$W = Ax^5 - Bx^4 - (C+D)x^3 + (E+F)x^2 \tag{25}$$

Shear Deformation Model

In this model, the coefficients in equation (25) are defined as

$$A = S_{55} \cdot A^* \quad (26\ a)$$
$$B = S_{55} \cdot B^* \quad b)$$
$$C = S_{55} \cdot C^* \quad c)$$
$$D = S_{33} \cdot D^* \quad d)$$
$$E = S_{55} \cdot E^* \quad e)$$
$$F = S_{33} \cdot F^* \quad f)$$

where

$A^* = 0.003704/20$; $B^* = 0.00615/20$, $C^* = 9.631/6$
$D^* = 0.02222/6$ $E^* = 194.5/2$; $F^* = 0.0123/2$

Classical Shear Deformation Model

In this model, the coefficients in equation (25) are defined as

$$A = A^*/C_{55} \quad (27\ a)$$
$$B = B^*/C_{55} \quad b)$$
$$C = C^*/C_{55} \quad c)$$
$$D = D^*/C_{33} \quad d)$$
$$E = E^*/C_{55} \quad e)$$
$$F = F^*/C_{33} \quad f)$$

The coefficients, A, B, C, D, E and F are seen in table 2 and 3 for shear deformation and classical shear deformation model respectively.

Classical Model

In this model, the coefficients A, B, C, E are the same coefficients as defined in equation (27) ; D and F vary.

RESULTS

C_{33} = 0.3071 E5	C_{25} = 0.3147 E5
C_{55} = 0.1327 E5	C_{36} = - 0.3147 E5
S_{33} = 0.4561 E-4	S_{55} = 0.1356 E-3

Table.1. Beam stiffnesses and beam compliance terms

A = 2.5113 E-8	C = 0.2176 E-3
B = 6.9495 E-8	E = 13.1871 E-3
D = 0.1689 E-8	F = 28.8051 E-8

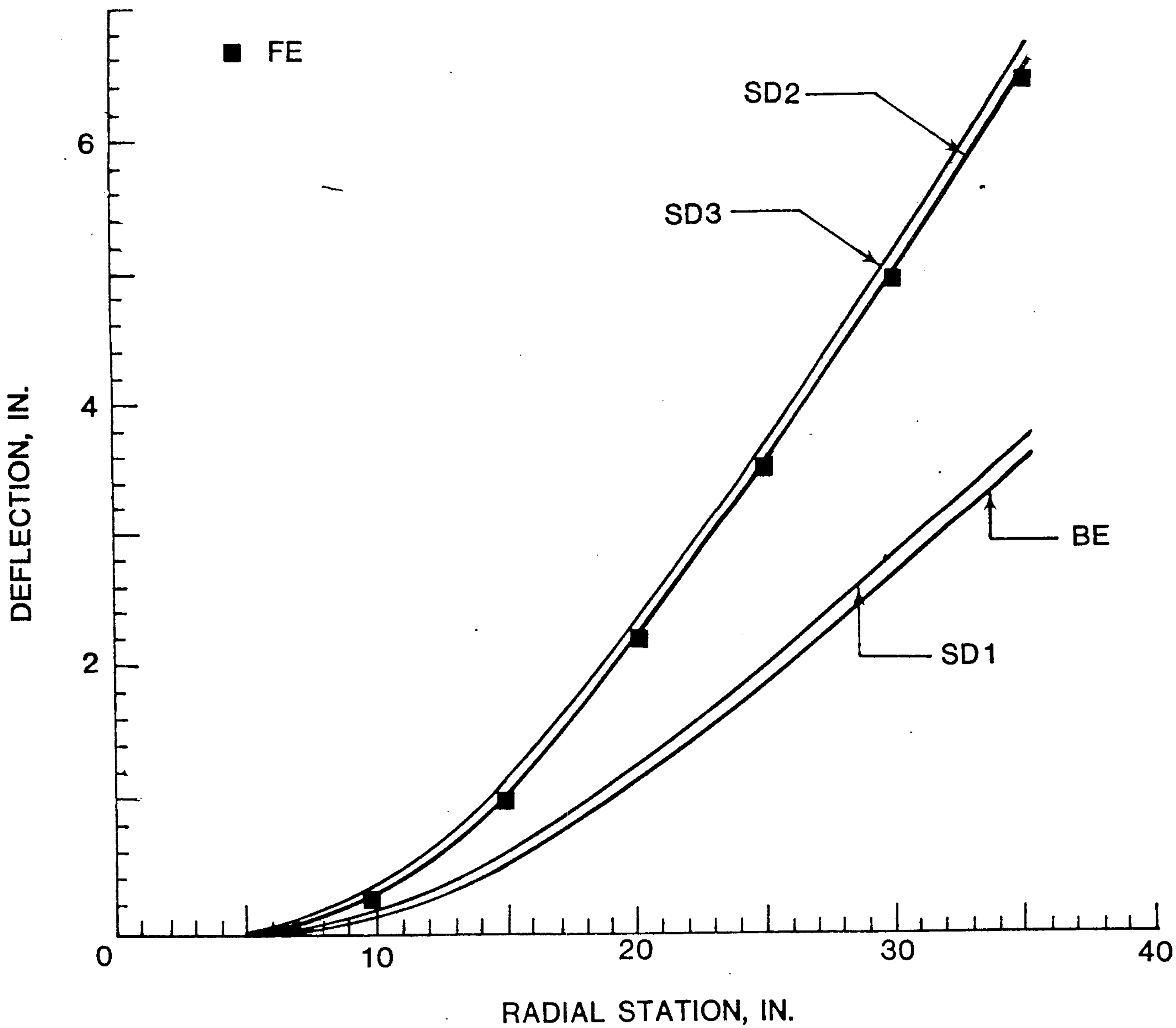
Table 2. Coefficients for shear deformation model

A = 1.3852 E-8	C = 0.1201 E-3
B = 3.8332 E-8	E = 7.2738 E-3
D = 0.1206 E-8	F = 20.0261 E-8

Table. 3. Coefficients for classical shear deformation model

	RADIAL STATION	5.23	10.23	15.23	20.23	25.23	[illegible]	[illegible]	[IN]
(SD3)	COMPLETE SHEAR DEFORMATION	0	0.3025	1.0961	2.2483	3.6035	5.0603	[illegible]	
FE	FINITE ELEMENT	0	0.30	1.09	2.23	3.58	4.99	6.53	
(SD1)	CLASSICAL SHEAR DEFORMATION	0	0.1669	0.6083	1.2445	1.9870	2.7902	3.6405	[IN]
(BE)	CLASSICAL	0	[illegible]	0.6063	[illegible]	[illegible]	[illegible]	[illegible]	
(SD2)	NON-CLASSICAL SHEAR DEFORMATION	0	0.3014	1.0949	2.2471	3.5828	5.0572	6.5448	

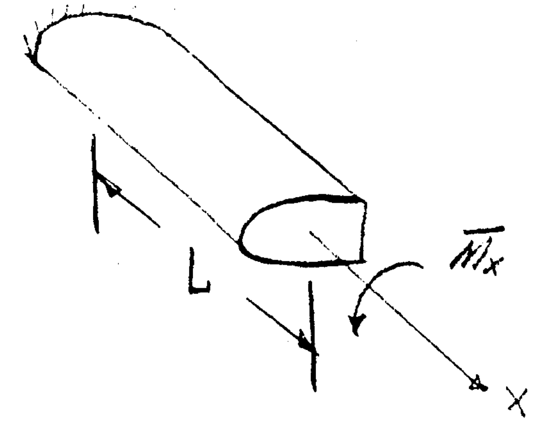
Table. 4. Beam deflection due to blade [illegible] lift for different models used.


FE
SD2
SD3
BE
SD1
DEFLECTION, IN.
6
4
2
0
10
20
30
40
RADIAL STATION, IN.

ATTACHMENT 2

Warping Effects in the Rotor Blade Model

Consider the root fixed and a tip torque applied.



$$\overline{M}_x = C_T \varphi_{,x} - C_{77} \varphi_{,xxx} \qquad (1)$$

where

$$C_T = C_{44} - \frac{(C_{14})^2}{C_{11}} \qquad (2)$$

At $x = 0$: $\varphi = 0$, $\varphi_{,x}(0) = 0$ (No warping)

The classical St. Venant solution is

$$\varphi_{CL} = \frac{\overline{M}_x}{C_T} x \qquad (5)$$

With restrained warping, the solution is found from

$$\varphi = \varphi_{CL} + \varphi' \qquad (6)$$

$$C_T \varphi' - C_{77} \varphi'_{,xx} = C_1 \quad \text{or}$$

$$\varphi'_{,xx} - p^2 \varphi' = - \frac{C_1}{C_{77}} \qquad (7)$$

Assume that only the decaying root will be used so that

$$\phi' = C_2 e^{-px} + C_1' \qquad (8)$$

Therefore

$$\phi = C_2 e^{-px} + C_1' + \frac{\bar{M}_x X}{C_T} \qquad (9)$$

$$\phi' = -pC_2 e^{-px} + \frac{\bar{M}_x}{C_T}$$

$$C_2 = \frac{\bar{M}_x}{pC_T} \qquad (10)$$

$$C_1' = - C_2 \qquad (11)$$

so

$$\boxed{\phi = \frac{\bar{M}_x}{C_T}\left[\frac{1}{p}(e^{-px}-1) + x\right]} \qquad (12)$$

Tip Deflection:

$$\phi(L) = \frac{\bar{M}_x}{C_T}\left(L - \frac{1}{p}\right) = \frac{\phi(L)}{CL}\left(1 - \frac{1}{Lp}\right) \quad ($$

where

$$p^2 = \frac{C_T}{C_{77}} \qquad (14.$$

CALCULATION OF TIP DEFLECTION

$$C_T = C_{44} - \frac{C_{14}^2}{C_{11}}$$

$$C_T = 0.49925\,E4 \qquad [lb\text{-}in^2]$$

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$$P^2 = \frac{C_T}{C_{77}}$$

$$C_{77} = 0.06645\,E5 \qquad [lb\text{-}in^4]$$

$$P^2 = 0.751317 \qquad [1/in^2]$$

$$P = 0.867 \qquad [1/in]$$

$$\frac{\varphi_\omega(L)}{\varphi_{CL}(L)} = 1 - \frac{1}{L \cdot P}$$

$$\frac{\varphi_\omega(L)}{\varphi_{CL}(L)} = 0.96$$

$$\varphi_{CL}(L) = \frac{\overline{M}_x \cdot L}{C_T} \cdot (57.3)$$

$$\varphi_{CL}(L) = 17.19$$

$$\varphi_\omega(L) = 16.50$$

ATTACHMENT 3

10 July 198?

TWIST UNDER EXTENSION
- Rotor Blade Model -

The axial force distribution is

$$N = 913.83 - 7.875x - 0.75287x^2 \quad (1a)$$

or

$$N = N_0 - N_1 x - N_2 x^2 \quad (1b)$$

We desired the twist with no torque applied.

$$(M_x)_{Total} = C_{11} U_{,x} + C_{44} \varphi_{,x} - C_{77} \varphi_{,xxx}$$

$$= 0 \quad (2)$$

$$N = C_{11} U_{,x} + C_{14} \varphi_{,x} \quad (3)$$

Eliminate $U_{,x}$ in (2)

$$U_{,x} = \frac{C_{77}}{C_{14}} \varphi_{,xxx} - \frac{C_{44}}{C_{14}} \varphi_{,x} \quad (4)$$

Substitute the above into (3)

$$N = \frac{C_{11}}{C_{14}} \left(C_{77} \varphi_{,xxx} - C_T \varphi_{,x} \right) \quad (5)$$

where $$C_T = C_{44} - \frac{C_{14}^2}{C_{11}}$$

Therefore

$$\varphi_{,xxx} - p^2\varphi_{,x} = \frac{C_{14}}{C_{11}C_{77}}\left(N_0 - N_1 x - N_2 x^2\right) \qquad (6)$$

where

$$p^2 = \frac{C_T}{C_{77}}$$

let say

$$K = \frac{C_{14}}{C_{11}C_{77}}$$

and integrate (6)

$$\varphi_{,xx} - p^2\varphi = K\left(N_0 x - \frac{N_1}{2}x^2 - \frac{N_2}{3}x^3 + k_1\right) \qquad (7)$$

Particular solution

$$\varphi_p = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad (8)$$

so

$$2a_2 + 6a_3 x - p^2\left(a_0 + a_1 x + a_2 x^2 + a_3 x^3\right) = K\left(N_0 x - \frac{N_1}{2}x^2 - \frac{N_2}{3}x^3 + k_1\right)$$

$$\begin{aligned} 6a_3 - p^2 a_1 &= K N_0 \\ -p^2 a_2 &= -\frac{K N_1}{2} \\ -p^2 a_3 &= -\frac{K N_2}{3} \\ -p^2 a_0 + 2a_2 &= K k_1 \end{aligned} \qquad (9)$$

The solution of system (9) is

$$a_0 = \frac{K}{p^2}\left(\frac{N_1}{p^2} - k_1\right) \qquad (10a)$$

$$a_1 = \frac{K}{p^2}\left(\frac{2N_2}{p^2} - N_0\right) \qquad b)$$

$$a_2 = \frac{KN_1}{2p^2} \qquad c)$$

$$a_3 = \frac{KN_2}{3p^2} \qquad d)$$

Therefore the particular solution is

$$\varphi_p = \frac{K}{p^2}\left\{\left(\frac{N_1}{p^2} - k_1\right) + \left(\frac{2N_2}{p^2} - N_0\right)x + \frac{N_1}{2}x^2 + \frac{N_2}{3}x^3\right\} \qquad (11)$$

The form of the solution is

$$\varphi = C_2 e^{-px} + \frac{K}{p^2}\left\{\left(\frac{N_1}{p^2} - k_1\right) + \left(\frac{2N_2}{p^2} - N_0\right)x + \frac{N_1}{2}x^2 + \frac{N_2}{3}x^3\right\} \qquad ($$

Boundary conditions

$$\varphi_{,x}(0) = 0 \qquad (13a)$$

$$-pC_2 + \frac{K}{p^2}\left\{\frac{2N_2}{p^2} - N_0\right\} = 0 \qquad b)$$

$$\varphi(0) = 0 \qquad (14a)$$

$$C_2 + \frac{K}{p^2}\left(\frac{N_1}{p^2} - k_1\right) = 0 \qquad b)$$

thus

$$C_2 = \frac{1}{p}\cdot\frac{K}{p^2}\left\{\frac{2N_2}{p^2} - N_0\right\} \qquad (15a)$$

and

$$k_1 = \frac{1}{p}\left\{\frac{2N_2}{p^2} - N_0\right\} - \frac{N_1}{p^2} \qquad b)$$

Therefore

$$\varphi = \frac{K}{p^2}\left\{\left(\frac{2N_2}{p^2} - N_0\right)\left[\frac{1}{p}\left(e^{-px} - 1\right) + x\right] + \frac{2N_1}{p^2} + \frac{N_1}{2}x^2 + \frac{N_2}{3}x^3\right\}$$

CALCULATION OF TIP DEFLECTION

$$\varphi_{CL}(L)\Big|_{L=30} = 14.9$$

$$\varphi_{\omega}(L)\Big|_{L=30} = 13.927$$

(Circle approach is used to determine C_{77})

$$\varphi_{\omega}(L)\Big|_{L=30} = 13.998$$

(Rectangular approach is used to find C_{77}

$$\varphi_{FEM}(L)\Big|_{L=30} \cong 14.$$

ATTACHMENT 4.

15 July 1986

STRAINS UNDER EXTENSION

CLASSICAL THEORY

The axial force distribution is

$$N = N_0 - N_1 x - N_1 x^2 \qquad (1)$$

If one desires the twist with no torque applied the following will be written

$$(M_x)_{Total} = C_{14} U_{,x} + C_{44} \varphi_{,x} = 0 \qquad (2)$$

so,

$$U_{,x} = -\frac{C_{44}}{C_{14}} \varphi_{,x} \qquad (3)$$

The axial strain is

$$(\varepsilon_{xx})_{CL} = U_{,x} + y\beta_{z,x} + z\beta_{y,x} \qquad (4)$$

and

$$M_y = M_z = 0 \qquad (5)$$

therefore

$$\beta_{y,x} = -\frac{C_{25}}{C_{55}} \gamma_{xy} \qquad (6)$$

and

$$\beta_{z,x} = -\frac{C_{36}}{C_{66}} \gamma_{xz} \qquad (7)$$

also

$$\gamma_{xy} = S_{22} Q_y + S_{25} M_y \tag{8}$$

and

$$\gamma_{xz} = S_{33} Q_z + S_{36} M_z \tag{9}$$

Since there is no shear force

$$Q_y = Q_z = 0 \tag{10}$$

With the aid of the equations (5) and (10)

$$\gamma_{xy} = \gamma_{xz} = 0 \tag{11}$$

therefore from equations (6) and (7)

$$\beta_{y,x} = \beta_{z,x} = 0 \tag{12}$$

thus equation (4) becomes

$$(\varepsilon_{xx})_{CL} = U_{,x} \tag{13}$$

Using equation (3)

$$(\varepsilon_{xx})_{CL} = -\frac{C_{44}}{C_{14}} \varphi_{,x} \tag{14}$$

From compliance relationship

$$\varphi_{,x} = S_{14} N \tag{15}$$

Using equation (1) and (15) into (14) yields

$$(\varepsilon_{xx})_{CL} = -\frac{C_{44}}{C_{14}} \cdot S_{14} \, (N_0 - N_1 x - N_2 x^2) \qquad (16)$$

The membrane shear strain is

$$(\gamma_{xs})_{CL} = \gamma_{xy} \frac{dy}{ds} + \gamma_{xz} \frac{dz}{ds} + \frac{2A_e}{c} \varphi_{,x} \qquad (17)$$

Since γ_{xy} and γ_{xz} is zero (equation 11)

$$(\gamma_{xs})_{CL} = \frac{2A_e}{c} \varphi_{,x} \qquad (18)$$

or

$$(\gamma_{xs})_{CL} = \frac{2A_e}{c} \cdot S_{14} \cdot (N_0 - N_1 x - N_2 x^2) \qquad (19)$$

COMPLETE BEAM THEORY

The form of the total torque is

$$(M_x) = C_{14} U_{,x} + C_{44} \varphi_{,x} - C_{77} \varphi_{,xxx} = 0 \qquad (20)$$

If one eliminates $U_{,x}$ in above

$$U_{,x} = \frac{C_{77}}{C_{14}} \varphi_{,xxx} - \frac{C_{44}}{C_{14}} \varphi_{,x} \qquad (21)$$

The axial strain is

$$(\varepsilon_{xx})_{CBT} = \frac{C_{77}}{C_{14}} \varphi_{,xxx} - \frac{C_{44}}{C_{14}} \varphi_{,x} + \psi \varphi_{,xx} \qquad (22)$$

The closed form solution of φ is

$$\varphi = \frac{K}{p^2}\left\{\left(\frac{2N_2}{p^2} - N_0\right)\left[\frac{1}{p}\left(e^{-px} - 1\right) + x\right] + \frac{2N_1}{p^2} + \frac{N_1}{2}x^2 + \frac{N_2}{3}x^3\right\} \quad (23)$$

so,

$$\varphi_{,x} = \frac{K}{p^2}\left\{\left(\frac{2N_2}{p^2} - N_0\right)\left(1 - e^{-px}\right) + N_1 x + N_2 x^2\right\} \quad (24)$$

$$\varphi_{,xx} = \frac{K}{p^2}\left\{\left(\frac{2N_2}{p^2} - N_0\right) p \cdot e^{-px} + N_1 + 2N_2 x\right\} \quad (25)$$

$$\varphi_{,xxx} = \frac{K}{p^2}\left\{\left(\frac{2N_2}{p^2} - N_0\right)\left(-p^2 e^{-px}\right) + 2N_2\right\} \quad (26)$$

Substitution of $\varphi_{,x}$, $\varphi_{,xx}$, $\varphi_{,xxx}$ into equation (22) gives

$$\begin{aligned}(\varepsilon_{xx})_{CBT} = {} & \frac{C_{77}}{C_{14}} \cdot \frac{K}{p^2}\left[\left(\frac{2N_2}{p^2} - N_0\right)\left(-p^2 e^{-px}\right) + 2N_2\right] \\ & - \frac{C_{44}}{C_{14}}\frac{K}{p^2}\left[\left(\frac{2N_2}{p^2} - N_0\right)\left(1 - e^{-px}\right) + N_1 x + N_2 x^2\right] \\ & + \Psi_m \frac{K}{p^2}\left[\left(\frac{2N_2}{p^2} - N_0\right)\left(p e^{-px}\right) + N_1 + 2N_2 x\right]\end{aligned} \quad (27)$$

where Ψ_m is the max value of warping function, which is

$$\Psi_m = -0.0464 \quad [\mathrm{IN}^2] \quad (28)$$

The membrane shear strain is

$$(\gamma_{xs})_{CBT} = \frac{2Ae}{C} \cdot \varphi_{,x} \tag{29}$$

or

$$(\gamma_{xs})_{CBT} = \frac{2Ae}{C} \cdot \frac{K}{p^2} \left\{ \left(\frac{2N_2}{p^2} - N_0\right)\left(1 - e^{-px}\right) + N_1 x + N_2 x^2 \right\} \tag{30}$$

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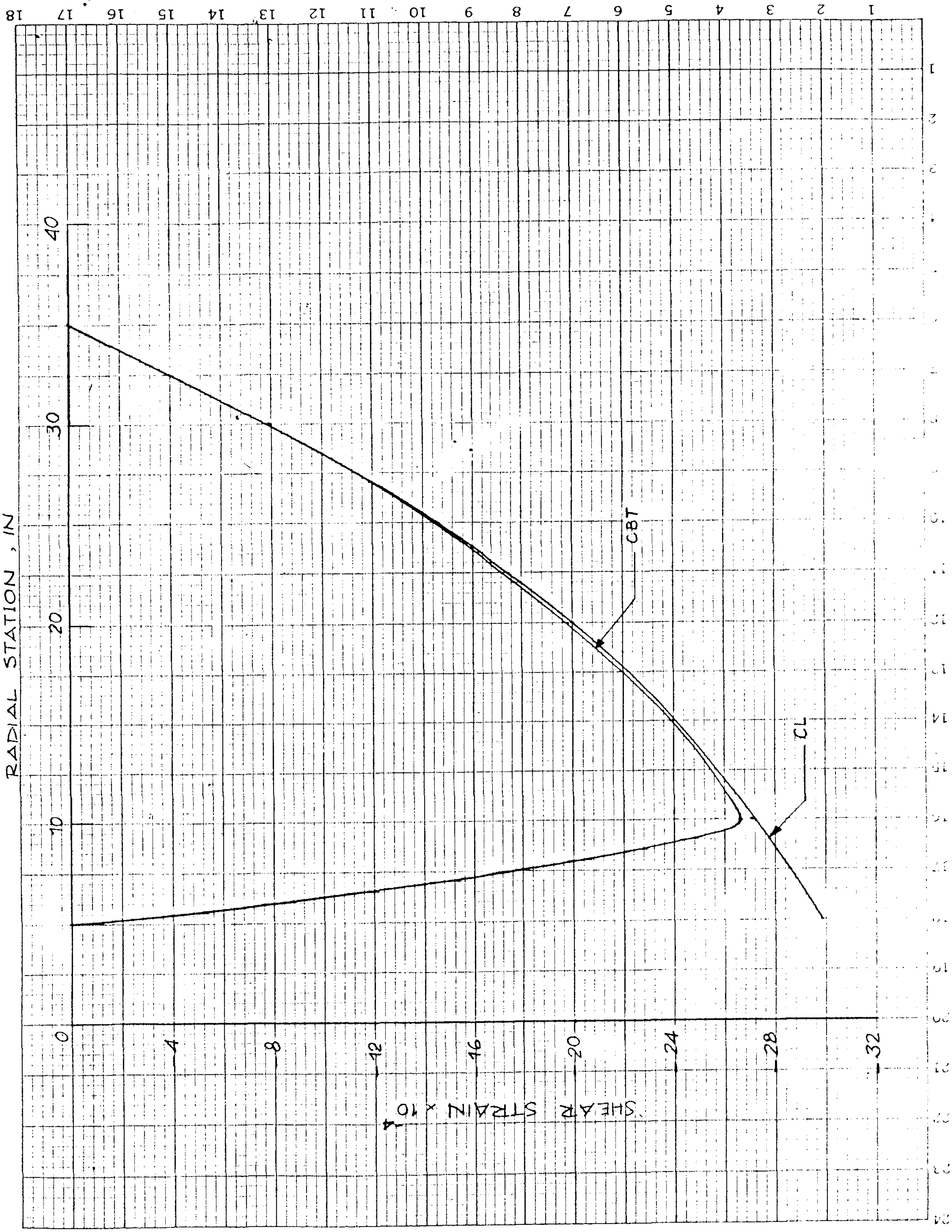

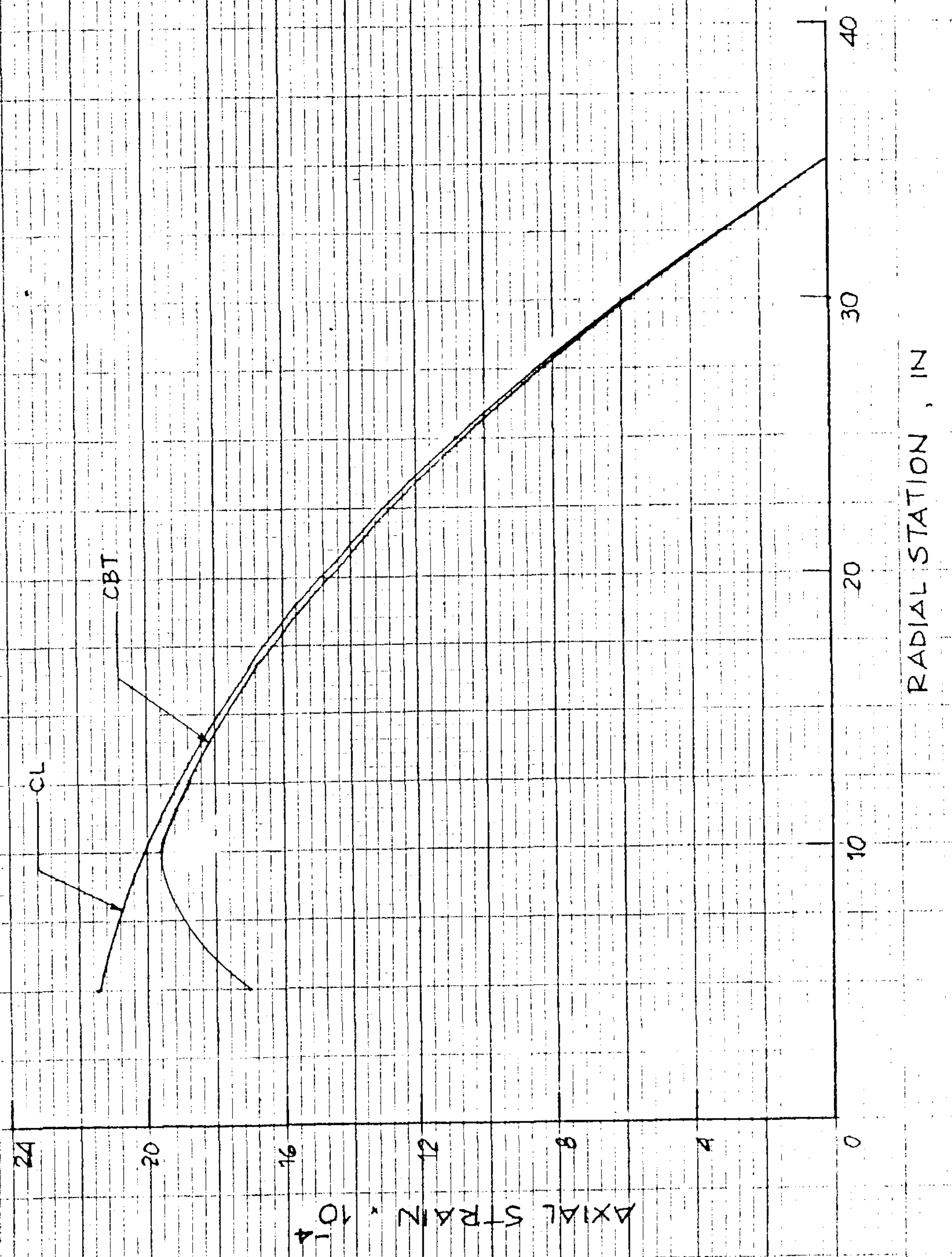
RADIAL STATION , IN
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ATTACHMENT 5

14 July 1986

WARPING EFFECTS ON ROTOR BLADE MODELS

- THE TORSION-RELATED WARPING FUNCTION -

The torsion-related warping function, ψ, is defined as

$$\psi(s) = \frac{2A_e}{c}\, s - 2\omega(s) \qquad (1)$$

where A_e is the enclosed area of the cross section, c is the circumference and

$$\omega(s) = \frac{1}{2}\int_0^s r_n\, ds \qquad (2)$$

which is the sectorial area swept out as s increases.

D spar cross sections

Co-ordinates of the D spar undertaken is shown in Fig.1. In order to calculate the warping function, one of the co.ordinate, say z, has to be found the function of the other one, y; to do so, the curve fitting method may be used. This analysis is summarized in Appendix I.

An other quick and useful approach to find the warping function is to consider the D spar as two region: one of them is rectangular the other one is a part of a circle. This approach is shown in Fig. 2.

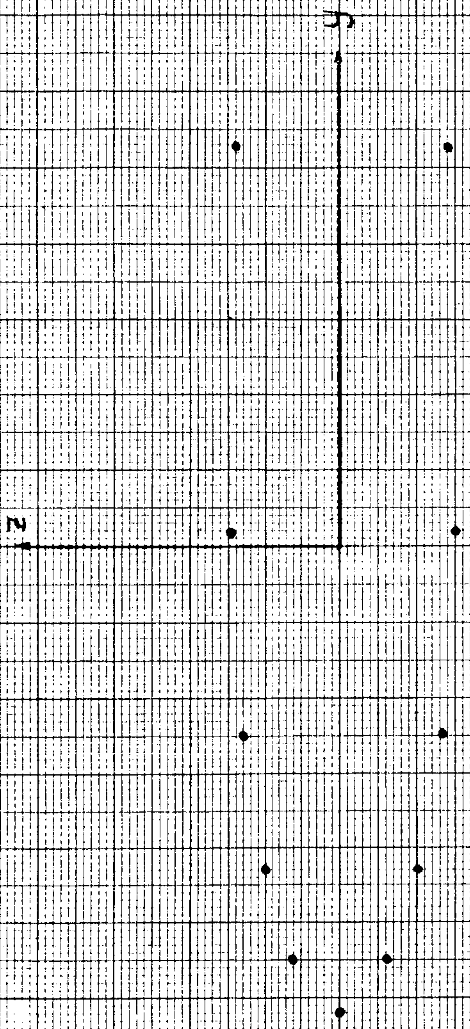

Fig. 1 Co-ordinates of the D spar

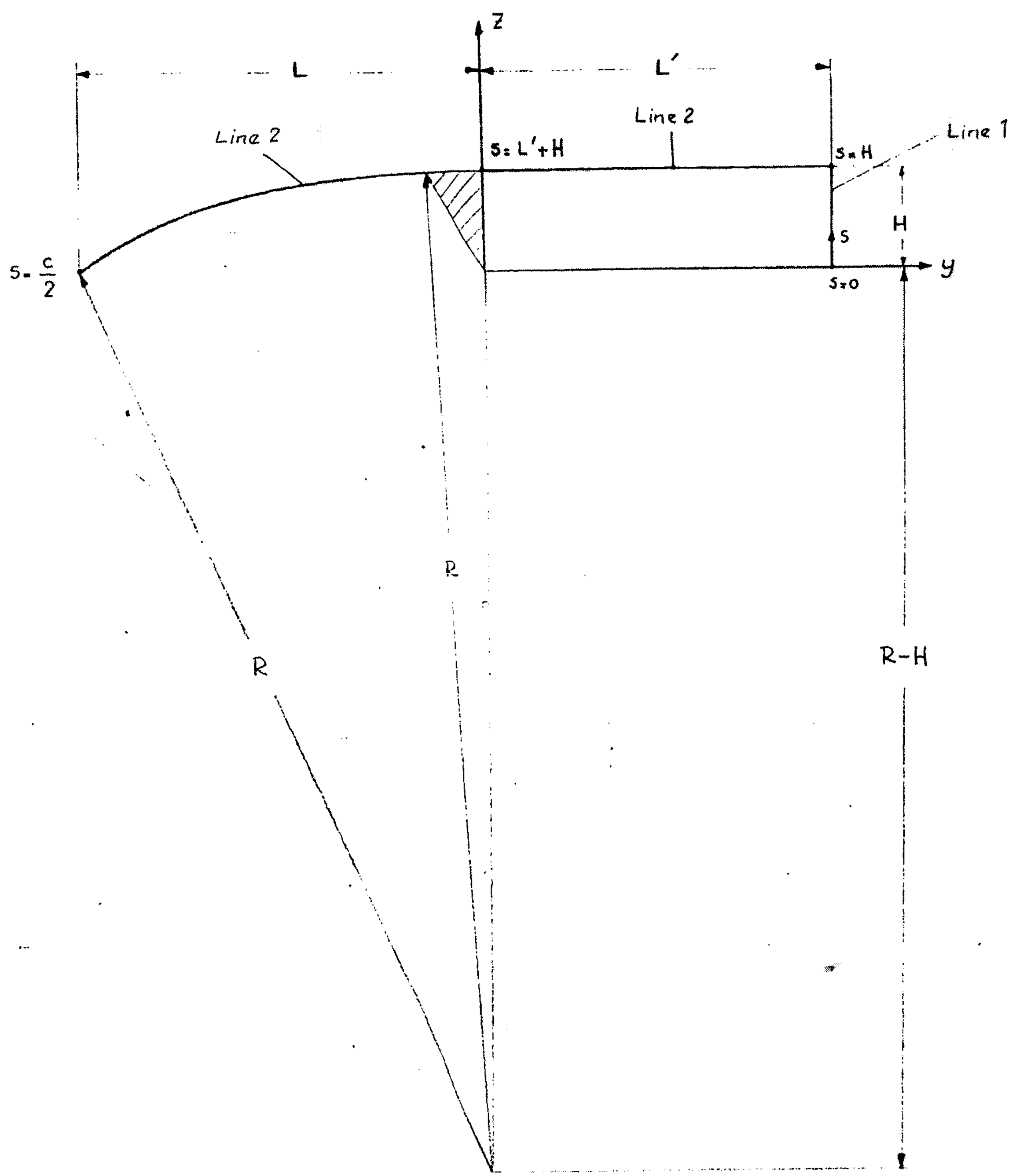

Fig. 2.

The radius of the circle considered is found to be as

$$R = \frac{H^2 + L^2}{2H} \tag{3}$$

To find the enclosed area the following method is used :

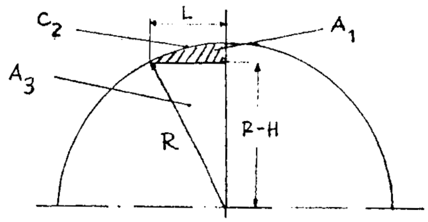

Fig.3.

The total area, $A_1 + A_3$, is

$$A_1 + A_3 = \frac{R^2}{2} \arcsin \frac{L}{R} \tag{4}$$

and A_3 is

$$A_3 = (R-H) \cdot \frac{L}{2} \tag{5}$$

therefore the area wanted, A_1, is

$$A_1 = \frac{R^2}{2} \arcsin \frac{L}{R} - (R-H) \cdot \frac{L}{2} \tag{6}$$

The area of the rectangular region, A_2, is

$$A_2 = L' \cdot H \tag{7}$$

thus, the total enclosed area of the D spar is

$$A_e = 2 (A_1 + A_2) \tag{8a}$$

or

$$A_e = 2\left[L'H + \frac{R^2}{2}\arcsin\frac{L}{R} - (R-H)\frac{L}{2} \right] \tag{8b}$$

The circumference of the circular part region, c_2, is

$$c_2 = R \arcsin\frac{L}{R} \tag{9}$$

The circumference of the rectangular region, c_1, is

$$c_1 = H + L' \tag{10}$$

then the total circumference of the D spar, c, is

$$c = 2(c_1 + c_2) \tag{11a}$$

or

$$c = 2\left(L' + H + R\arcsin\frac{L}{R} \right) \tag{11b}$$

Therefore $\frac{2A_e}{c}$, let say

$$B = \frac{2A_e}{c} \tag{12}$$

so

$$B = \frac{L'H + \frac{R^2}{2}\arcsin\frac{L}{R} - (R-H)\frac{L}{2}}{L' + H + R\arcsin\frac{L}{R}} \tag{13}$$

Now, the warping function can be rewritten as

$$\psi(s) = Bs - 2\omega(s) \tag{14}$$

Since the main purpose is to find the warping stiffness, C_{77}, which is calculated by using a line integral :

$$C_{77} = \oint K_{11} \cdot \Psi^2 \, ds \tag{15}$$

and, now, let consider the lines shown in Fig 2. are made out of different materials ; therefore equation (15) becomes (using symmetr

$$\frac{1}{2} C_{77} = \int_0^H K_{11}^{(1)} \Psi_1^2 \, ds + \int_H^{L'+H} K_{11}^{(2)} \Psi_2^2 \, ds + \int_{L'+H}^{C/2} K_{11}^{(3)} \Psi_3^2 \, ds \tag{16}$$

where

$$\Psi_1 = Bs - 2\omega_1(s) \qquad 0 \leq s \leq H \tag{17}$$

$$\Psi_2 = Bs - 2\omega_2(s) \qquad H \leq s \leq L'+H \tag{18}$$

$$\Psi_3 = Bs - 2\omega_3(s) \qquad L'+H \leq s \leq \frac{c}{2} \tag{19}$$

so, we reduced the problem which is to find the sectorial areas swept out for each region.

For Line 1

$$r_n = L' \tag{20}$$

so

$$2\omega_1(s) = \int_0^s L' ds \tag{21}$$

then

$$2\omega_1(s) = L's \qquad 0 \leq s \leq H \tag{22}$$

For line 2

$$r_n = H \tag{23}$$

so

$$2\omega_2(s) = L'H + \int_0^s H ds \tag{24}$$

then

$$2\omega_2(s) = L'H + Hs \qquad H \leq s \leq L'+H \tag{25}$$

For line 3, we have to find the shaded area, A', as function of s. The form of A' is found by substracting the area below the shaded area from the area of the circular part. One obtains the A' as

$$A' = \frac{1}{2}\left[Rs - (R-H)\, R \sin\frac{s}{R} \right] \tag{26}$$

therefore

$$2\omega_3(s) = 2L'H + Rs - (R-H)\, R \sin\frac{s}{R} \qquad L'+H \le s \le \frac{c}{2} \tag{27}$$

Using equations (17) and (22) gives ψ_1,

$$\psi_1(s) = (B - L')s \qquad 0 \le s \le H \tag{28}$$

Equations (18) and (25) gives ψ_2,

$$\psi_2(s) = (B - H)s - L'H \qquad H \le s \le L'+H \tag{29}$$

and equations (19) and (27) gives ψ_3,

$$\psi_3(s) = (B - R)s + (R-H)R \sin\frac{s}{R} - 2L'H \qquad L'+H \le s \le \frac{c}{2} \tag{30}$$

Equation (16) can be written as

$$C_{77} = 2\left(K_{11}^{(1)} I_1 + K_{11}^{(2)} I_2 + K_{11}^{(3)} I_3 \right) \tag{31}$$

where

$$I_1 = \int_0^H [(B - L')s]^2\, ds \tag{32a}$$

$$I_1 = \int_0^H (B-L')^2 s^2\, ds \; . \; K_{11}^{(1)} \; \frac{(B-L')^2}{3} s^3 \Big|_0^H \tag{32b}$$

or

$$\boxed{I_1 = \frac{(B-L')^2}{3} \cdot H^3} \tag{33}$$

and

$$I_2 = \int_H^{L'+H} \left[(B-H)s - L'H \right]^2 ds \tag{34a}$$

$$I_2 = \int_H^{L'+H} \left[(B-H)^2 s^2 - 2(B-H)L'Hs + L'^2H^2 \right] ds \tag{34b}$$

$$= \left[\frac{(B-H)^2}{3} s^3 - (B-H)L'H\, s^2 + L'^2 H^2 s \right] \Big|_H^{L'+H}$$

so

$$\boxed{I_2 = \left\{ \frac{(B-H)^2}{3} L'(L'^2 + 3L'H + 3H^2) - (B-H)L'^2 H(L' + 2H) + L'^3 H^2 \right\}} \tag{35}$$

and

$$I_3 = \int_{L'+H}^{c/2} \left[(B-R)s + (R-H)R \sin\frac{s}{R} - 2L'H \right]^2 ds \tag{36a}$$

$$I_3 = \int_{L'+H}^{c/2} \Big[(B-R)^2 s^2 + (R-H)^2 R^2 \sin^2\frac{s}{R} + 4L'^2H^2 + 2(B-R)(R-H)Rs\sin\frac{s}{R}$$

$$- 4(R-H)RL'H\sin\frac{s}{R} - 4(B-R)L'Hs \Big] ds \tag{36b}$$

$$I_3 = \left[\frac{(B-R)^2}{3} s^3 + (R-H)^2 R^2 \left(\frac{1}{2} s - \frac{R}{4} \sin \frac{2s}{R} \right) + 4L'^2 H'^2 s \right.$$

$$+ 2(B-R)(R-H)R \left(R^2 \sin \frac{s}{R} - s\, R \cos \frac{s}{R} \right)$$

$$\left. - 4(R-H)RL'H \left(-R\cos\frac{s}{R} \right) - 2(B-R)L'Hs^2 \right] \Bigg|_{L'+H}^{C/2} \qquad (36c)$$

$$I_3 = \left\{ \frac{(B-R)^2}{3} \left[\frac{C^3}{8} - (L'+H)^3 \right] + (R-H)^2 R^2 \left\{ \frac{1}{2} \left(\frac{C}{2} - L' - H \right) - \frac{R}{4} \left[\sin \frac{C}{R} - \sin \frac{2(L}{} \right. \right. \right.$$

$$+ 4L'^2 H'^2 \left(\frac{C}{2} - L' - H \right)$$

$$+ 2(B-R)(R-H)R \left\{ R^2 \left(\sin \frac{C}{2R} - \sin \frac{L'+H}{R} \right) - \left[\frac{C}{2} R \cos \frac{C}{2R} - (L'+H) R \cos \frac{L'+H}{R} \right. \right.$$

$$\left. + 4(R-H)RL'H \left(R\cos\frac{C}{2R} - R\cos\frac{L'+H}{R} \right) - 2(B-R)L'H \left[\frac{C^2}{4} - (L'+H)^2 \right] \right\} ($$

so we know I_1 (equation (33)), I_2 (equation (35)) and I_3 (equation (37)).

If the D spar is made out of the same material

$$K_{11}^{(1)} = K_{11}^{(2)} = K_{11}^{(3)} = K_{11} \qquad (38)$$

then

$$C_{77} = 2K_{11} \left(I_1 + I_2 + I_3 \right) \qquad (39)$$

The Equivalent Rectangular Area Approach

Let consider a rectangular cross section which has the same enclosed area as the D spar.

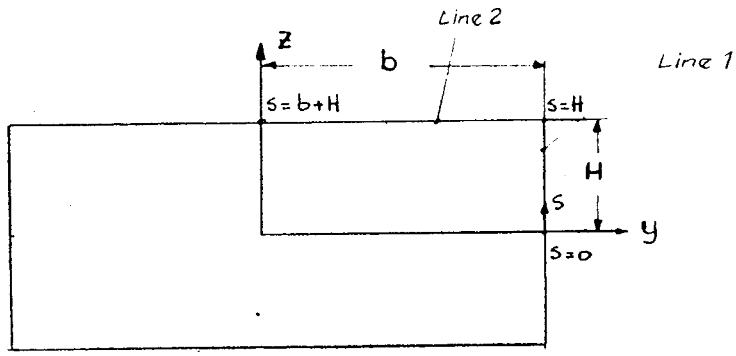

Fig. 4.

It is presumed that the rectangular has the same width, H, as the D spar; therefore

$$(A_e)_{D\ spar} = 4bH \tag{40a}$$

or

$$b = \frac{(A_e)_{D\ spar}}{4H} \tag{40b}$$

so, the circumference of the rectangular cross section is

$$(C)_{rec} = 4(b+H) \tag{41}$$

then the ratio of $\frac{2A_e}{C}$, B',

$$B' = \frac{2bH}{b+H} \tag{42}$$

The form of the warping function, Ψ_R, is

$$\Psi_R = B's - 2\omega(s) \tag{43}$$

We will follow the same procedure as what we have done for D spar case. Let say, Line 1 and Line 2 are made out of different materials. Using symmetry gives

$$C_{77}^{R} = 4\left\{\int_0^H K_{11}^{(1)} \Psi_{R1}^2 \, ds + \int_H^{b+H} K_{11}^{(2)} \Psi_{R2}^2 \, ds\right\} \tag{44}$$

where

$$\Psi_{R1} = B's - 2\omega_1(s) \qquad 0 \le s \le H \tag{45}$$

$$\Psi_{R2} = B's - 2\omega_2(s) \qquad H \le s \le b+H \tag{46}$$

For Line 1

$$r_n = b \tag{47 a}$$

so

$$2\omega_1(s) = \int_0^s b \, ds \tag{b}$$

$$2\omega_1(s) = bs \qquad 0 \le s \le H \tag{48}$$

For Line 2

$$r_n = H \tag{49 a}$$

so

$$2\omega_2(s) = bH + \int_0^s H \, ds \tag{b}$$

$$2\omega_2(s) = bH + Hs \qquad H \le s \le H+b \tag{50}$$

Using equations (45) and (48) gives

$$\Psi_{R1} = (B' - b)s \qquad 0 \leq s \leq H \tag{51}$$

Equations (46) and (50) gives

$$\Psi_{R2} = (B' - H)s - bH \qquad H \leq s \leq b + H \tag{52}$$

Equation (44) can be rewritten as

$$C_{77}^{R} = 4\left(K_{11}^{(1)} J_1 + K_{11}^{(2)} J_2\right) \tag{53}$$

where

$$J_1 = \int_0^H \left[(B' - b)s\right]^2 ds \tag{54}$$

$$J_1 = \int_0^H (B' - b)^2 s^2 \, ds = \frac{(B' - b)^2}{3} \cdot s^3 \Big|_0^H$$

$$\boxed{J_1 = \frac{(B' - b)^2}{3} H^3} \tag{55}$$

and

$$J_2 = \int_H^{b+H} \left[(B' - H)s - bH\right]^2 ds \tag{56}$$

$$J_2 = \int_H^{b+H} \left[(B' - H)^2 s^2 - 2(B' - H)bHs + b^2 H^2\right] ds$$

$$= \left[\frac{(B' - H)^2}{3} s^3 - (B' - H)bHs^2 + b^2 H^2 s\right] \Big|_H^{b+H}$$

so

$$J_2 = \frac{(B' - H)^2}{3}\left[(H + b)^3 - H^3\right] - (B' - H)bH\left[(b + H)^2 - H^2\right] + b^3 H^2$$

or

$$\boxed{J_2 = \frac{(B'-H)^2}{3}\left[(H+b)^3 - H^3\right] - (B'-H)bH\left[(b+H)^2 - H^2\right] + b^3H^2} \tag{57}$$

If

$$K_{11}^{(1)} = K_{11}^{(2)} = K \tag{58}$$

then

$$C_{77}^{R} = 4K_{11}(J_1 + J_2) \tag{59}$$

AN APPLICATION

In our case, the parameters are

$L' = 0.5265$; $H = 0.1469$; $L = 0.6210$; $R = 1.3861$ [IN]

then the enclosed area, A_e, and the circumference, c, of D spar are

$A_e = 0.2776$ [IN^2] ; $C = 2.6345$ [IN]

therefore

$$B = 0.2107 \quad [IN]$$

For [+20, -70, +20, -70, -70, +20] Graphite /Epoxy

$$K_{11} = 0.3139\ E6$$

and

$$C_{77} = 0.06645\ E5 \quad [lb\text{-}in^4]$$

Using the equivalent rectangular area approach

$H = 0.1469$; $b = 0.4724$ [IN]

and

$A_e = 0.2776$ [IN^2] $C = 2.4772$ [IN]

so

$$B' = 0.2241 \quad [IN]$$

and

$$C_{77}^{R} = 0.05964\ E5 \quad [lb\text{-}in^4]$$

The error produced between C_{77} and C_{77}^{R} is $\sim 10.3\%$ but, for instance twist tip deflection due to applied torque case the error produced is $\sim 0.3\%$.